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Dual solutions for second-order slip flow and heat transfer on a vertical permeable shrinking sheet

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KEYWORDS

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Abstract The aim of the paper is to study viscous fluid flow and heat transfer with second-order slip at linearly shrinking isothermal sheet in a quiescent medium. The sheet is permeable and subjected to constant suction. The governing equations consisting of the continuity, momentum, and the energy are transformed into a system of ordinary differential equations using suitable similarity transformation and solved numerically using the Runge–Kutta fourth-order method with the shooting technique. It is found that the problem possesses a dual physical solution. The effects of different parameters on the velocity and the temperature distributions as well as the skin-friction coefficient and the Nusselt number are presented graphically and in tabular form.

1. Introduction

A wide range of applications of nano-technology and micro-electro-mechanical systems have given a fillip to research area where a non-continuum behavior is present. In the present context, we are interested in studying surface-fluid interaction where slip flow regime occurs. In this regard, Kundsen number (K_n) is a deciding factor, which is a measure of molecular mean free path to characteristic length. When Kundsen number is very small, no slip is observed between the surface and the fluid and is in tune with the essence of continuum mechanics. However, when Kundsen number lies in the range 10^{-3} to 0.1, slip occurs at the surface-fluid interaction and is generally studied under the light of model Maxwell–Smoluchowski first-order slip boundary conditions. Adding, slip flow theory is an asset that enables to exploit Navier–Stokes equation even when the characteristic length approaches molecular mean free path. Slip flow theory has been validated by asymptotic solution of Boltzmann equation. In this analysis, inner kinetic solution is matched with outer (i.e., bulk) Navier–Stokes solution and the matching is obtained only when slip/jump coefficient are considered at boundary or at the surface (Hadjiconstantinou [1,2]). First and second slip flow coefficients are therefore outcomes of above said analysis. It is important to point that efficient slip flow model is always preferred, because of inherent simplicity over solution obtained for Boltzmann equation.

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The applicability of first-order slip model deteriorates as Kundsen is around or greater than 0.1. Therefore, a number of researchers have proposed second-order slip flow model. Wu [3] has proposed a second-order slip flow model for the flow of rarefied fluid along the surface based on numerical simulation of linearized Boltzmann equation. It is to mention in the light of above discussion that that either no slip regime or slip regime is taken as a boundary condition, the Navier–Stokes equation is still valid.

The flow induced by a moving surface has its importance in industrial applications and thus has been observed considerably in the literature. The permeable stretching/shrinking sheet is one such example, which has been studied with no slip regime or slip regime at the surface. Sakiadis [4], Tsou et al. [5], Crane [6], Chen and Char [7] contributed to the study of fluid flow along a moving/stretching surface. Magyari and Keller [8] presented an exact solution of self-similar boundary layer flow along a permeable stretching sheet. Andersson [9] obtained a solution of flow along stretching sheet with slip. Wang [10] analyzed the partial slip flow on stretching sheet in a quiescent medium. Miklavcic and Wang [11] studied flow on a shrinking sheet and argued the existence and non-uniqueness of a solution. Zhang et al. [12] obtained similarity solution of steady gaseous flow between parallel plates with first and second-order slip. Fang and Zhang [13] obtained an exact solution of MHD flow along a horizontal shrinking sheet without slip. Fang et al. [14] obtained an analytical solution of MHD flow along a shrinking sheet with first-order slip flow. Fang and Aziz [15] presented exact solutions of Navier–Stokes equation for second slip flow along a permeable stretching sheet. Yacob and Ishak [16] studied stagnation point flow of micropolar fluid over a shrinking sheet with a convective boundary condition. Rashidi et al. [17] studied flow of a second-grade fluid over a stretching or shrinking sheet using the multi-step differential transform method. Lok et al. [18] considered MHD flow along a shrinking sheet and inferred the existence of a dual solution for a small magnetic field. Fang et al. [19] presented an analytical solution for viscous flow along a shrinking sheet considering the second-order slip model presented by Wu [3]. Nanadeppanavar et al. [20] analyzed second-order slip flow over a horizontal shrinking sheet with a non-linear Navier boundary condition. Turkyilmazoglu [21] analytically studied heat and mass transfer in MHD viscous flow with hydrodynamic and thermal first-order slip over a stretching sheet for different thermal boundary condition. Turkyilmazoglu [22] presented analytically the existence of dual and triple solution in the flow of MHD viscoelastic fluid over a shrinking surface with first-order slip. Recently, Turkyilmazoglu [23] obtained analytically dual solution for MHD viscous flow with second-order hydrodynamic slip over a stretching/shrinking surface for non-isothermal/prescribed heat flux boundary condition.

In the present paper, we study viscous fluid flow and heat transfer with a second-order slip on a vertical isothermal shrinking sheet in a quiescent medium. The model of a second-order slip is the same as proposed by Wu [3].

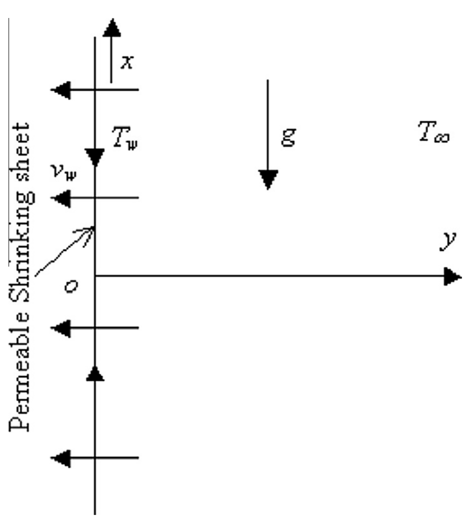

Figure 1 Problem schematic and coordinate system.

2. Formulation of the problem

Consider a continuous permeable vertical shrinking sheet at a temperature T_w with a linear velocity, $-cx$ ($c > 0$), and mass transfer velocity at the surface equal to v_w, in quiescent fluid of temperature T_∞. The x-axis is taken along the plate and the y-axis is taken perpendicular to the plate as is shown in Fig. 1. The fluid experiences a second-order slip at the sheet surface.

The governing equations of steady boundary layer flow are based on the continuity, momentum and the energy equations, which are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \pm g\beta(T - T_\infty), \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocities along x and y directions, respectively, g is the acceleration due to gravity, β is coeffi-

Table 1 Comparison of $f''(0)$ with Fang et al. [19].

	Fang et al. [19]		Present paper	
	$S = 2.0$, $\lambda = 0.1$, $\delta = -1.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -2.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -1.0$, $Gr = 0.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -2.0$, $Gr = 0.0$
First Solution (F.S.)	0.34115	0.2037	0.3412	0.2038
Second Solution (S.S.)	0.3159	0.2655	0.3158	0.2656

Table 2 Numerical values of $f'(0)$, $f''(0)$, and $\theta'(0)$ for different values of parameters.

	First solution (F.S.)			Second solution (S.S.)		
	$f'(0)$	$f''(0)$	$-\theta'(0)$	$f'(0)$	$f''(0)$	$-\theta'(0)$
S	$\lambda = 0.1$, $\delta = -1.0$, $Gr = 1.0$, $Pr = 1.0$					
0.8	−0.163636	0.95904	0.902876	−1.27753	1.505066	0.414859
1.0	−0.089994	0.834640	1.088547	−1.458002	1.516153	0.447765
1.5	−0.0172655	0.614395	1.551102	−1.785693	1.501880	0.611405
2.0	−0.000840	0.475790	2.027726	−2.003283	1.433267	0.893461
2.5	0.001989	0.385382	2.515598	–	–	–
λ	$S = 1.0$, $\delta = -1.0$, $Gr = 1.0$, $Pr = 1.0$					
0.1	−0.089994	0.834640	1.088547	−1.458002	1.516153	0.447765
0.5	0.016907	0.678128	1.120563	−1.573929	1.268885	0.325868
1.0	0.097625	0.553577	1.143890	−1.624939	1.007744	0.233873
1.5	0.150434	0.469225	1.158783	−1.634790	0.815099	0.179075
2.0	0.188044	0.407801	1.169221	−1.629725	0.675426	0.144480
δ	$S = 1.0$, $\lambda = 0.1$, $Gr = 1.0$, $Pr = 1.0$					
−1.0	−0.089994	0.834640	1.088547	−1.458002	1.516153	0.447765
−1.5	0.036197	0.648852	1.126202	−1.3990641	1.585629	0.496185
−2.0	0.107318	0.538263	1.146645	−1.370171	1.611699	0.518061
−2.5	0.152608	0.465704	1.159391	−1.353232	1.624951	0.530418
−3.0	0.183897	0.414629	1.168077	−1.342146	1.632879	0.538337
Gr	$S = 3.0$, $\lambda = 0.1$, $\delta = -1.0$, $Pr = 1.0$					
−1.0	−0.212439	0.268610	2.953389	−1.476949	0.545133	2.436877
−2.0	−0.344255	0.249760	2.917468	−1.500669	0.564947	2.465038
1.0	0.00200	0.323229	3.009358	–	–	–
5.0	0.333050	0.465797	3.090795	−4.463322	5.309009	1.435054
10.0	0.656950	0.673721	3.165763	−5.823595	9.384085	1.398844
Pr	$S = 1.0$, $\lambda = 0.1$, $\delta = -1.0$, $Gr = 1.0$					
0.5	0.022133	0.929657	0.650940	−1.540567	1.666165	0.302569
0.72	−0.033306	0.879821	0.847457	−1.500010	1.590934	0.366216
1.0	−0.089994	0.834640	1.088547	−1.458002	1.516153	0.447765
3.0	−0.403212	0.690334	2.702276	−1.191716	1.116791	1.392029
5.0	–	–	–	–	–	–

cient of thermal expansion, v is the kinematic viscosity, ρ is the fluid density, C_p is the specific heat at constant pressure, T is the fluid temperature, and κ is the thermal conductivity of fluid.

The boundary conditions for this problem are given by

$$\begin{aligned} &y = 0: \; u = -cx + U_{slip}, \quad v = v_w, \quad T = T_w, \\ &y \to \infty: \; u = 0, T = T_\infty. \end{aligned} \tag{4}$$

Following the Wu [3] model, U_{slip} takes the form

$$\begin{aligned} U_{slip} &= \frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha} - \frac{3}{2}\frac{1-l^2}{K_n}\right)\gamma\frac{\partial u}{\partial y} \\ &\quad - \frac{1}{4}\left(l^4 + \frac{2}{K_n^2}(1-l^2)\right)\gamma^2\frac{\partial^2 u}{\partial y^2} \\ &= A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{5}$$

where $l = \min\left(\frac{1}{K_n}, 1\right), (\alpha, (0 \leqslant \alpha \leqslant 1)$ is the momentum accommodation coefficient and $\gamma\ (>0)$ is the mean free path. Hence, for any Kundsen number (K_n), A is positive and B is negative.

3. Method of solution

Here, we introduce the stream function $\psi\ (x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \tag{6}$$

where $\psi = x\sqrt{v, c}, f(\eta), \eta = y, \sqrt{\frac{c}{v}}$ is the similarity variable and $\theta = \frac{T-T_\infty}{T_w-T_\infty}$ is dimensionless fluid temperature.

It is observed that Eq. (1) is identically satisfied by $\psi\ (x, y)$. Substituting Eq. (6) into Eqs. (2) and (3), the resulting non-linear coupled ordinary differential equations are the following:

$$f''' + ff'' - f'^2 \pm Gr\theta = 0, \tag{7}$$

and

$$\theta'' + Prf\theta' = 0, \tag{8}$$

where a prime means a derivative with respect to η, f is the dimensionless stream function, $Gr\left\{= \frac{g\beta(T_w-T_\infty)x^3}{v^2, Re^2}\right\}$ is the buoyancy parameter, $Re\left\{= \frac{cx^2}{v}\right\}$ is the Reynolds number, and $Pr\left\{= \frac{\mu, C_p}{\kappa}\right\}$ is the Prandtl number. It should be noted that Gr

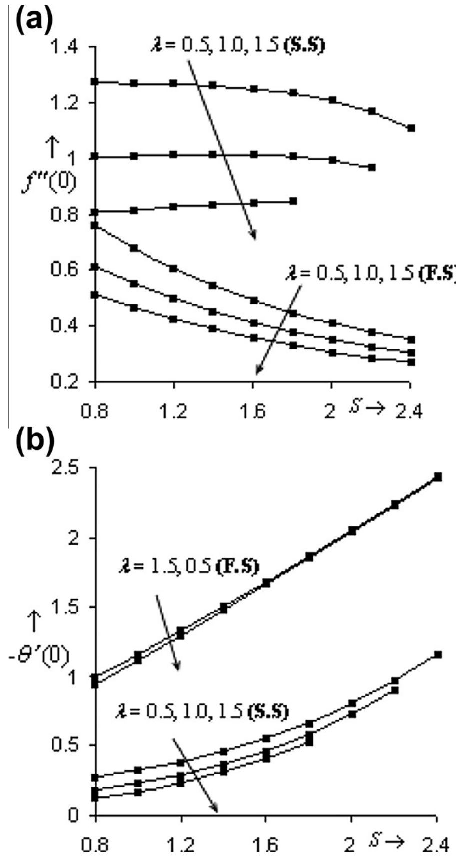

Figure 2 (a) Plot of $f''(0)$ versus S when $\delta = -1.0$ $Pr = 1.0$ and $Gr = 1.0$. (b) Plot of $-\theta'(0)$ versus S when $\delta = -1.0$ $Pr = 1.0$ and $Gr = 1.0$.

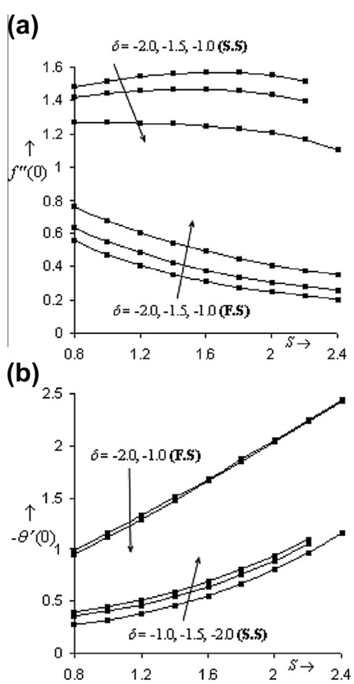

Figure 3 (a) Plot of $f''(0)$ versus S when $\lambda = 0.5$ $Pr = 1.0$ and $Gr = 1.0$. (b) Plot of $f''(0)$ versus S when $\lambda = 0.5$ $Pr = 1.0$ and $Gr = 1.0$.

is positive for assisting flow ($T_w > T_\infty$) and negative for opposing flow ($T_w < T_\infty$).

The corresponding boundary conditions, following (4) and (5), become

$$f(0) = S, f'(0) = -1 + \lambda f''(0) + \delta f'''(0),$$
$$\theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \tag{9}$$

where $S = \frac{-v_w}{\sqrt{v_f c}}, \lambda\left\{= A\sqrt{\frac{c}{v}}\right\}$ is the first-order slip parameter where $\lambda > 0$ while $\delta\left\{= \frac{B.c}{v}\right\}$ is the second-order slip parameter where $\delta < 0$. It important to point out that $S > 0$ for suction and $S < 0$ for injection. In the present study, we take only suction because in the case of a shrinking surface, suction helps retaining the vorticity within the boundary layer and laminar flow is thus maintained.

The expressions for the skin-friction coefficient C_f and the Nusselt number (Nu) are given by:

$$C_f = 2(Re)^{-1/2} f''(0), \quad Nu = -(Re)^{-1/2}\theta'(0). \tag{10}$$

The system of Eqs. (7) and (8) along with (9) is numerically solved using the Runge–Kutta fourth-order method with the shooting technique. The results obtained in particular cases are compared with the results presented by Fang et al. [18]. Table 1 shows the good agreement, which in turn vindicates the use of present numerical scheme.

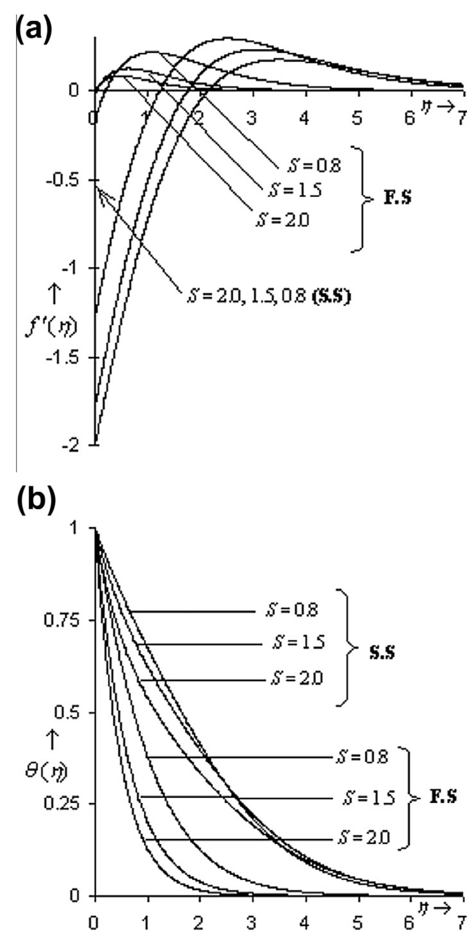

Figure 4 (a) Velocity distribution versus η when $Pr = 1.0$, $\lambda = 0.1$, $\delta = -1.0$ and $Gr = 1.0$. (b) Temperature distribution versus η when $Pr = 1.0$, $\lambda = 0.1$, $\delta = -1.0$ and $Gr = 1.0$.

4. Results and discussion

The system of Eqs. (7) and (8) along with (9) is numerically solved, and it is found that it possesses dual solution. Table 2 presents two physical values of each of $f'(0)$, $f''(0)$, and $-\theta'(0)$ for the same parametric values and the solutions are classified as First Solution (F.S.) and Second Solution (S.S.). $f'(0)$ measures the fluid velocity at surface, $f''(0)$ measures the skin friction, and $-\theta'(0)$ measures the rate of heat transfer. Similarly, each figure from 4 to 8 presents two sets of profiles of velocity and temperature, classified as F.S. and S.S. It is seen from Table 2, in the case of first solution, with the increase in parameter S (i.e., suction), λ and δ, the fluid velocity at surface increases while the skin friction decreases. It is seen that with the increase in the parameters S, λ and δ, the rate of heat transfer increases. For the second solution, with the increase in parameters S, λ and δ, the fluid velocity at surface and the skin friction decrease while the rate of heat transfer increases. Table 2 depicts that for the first solution, with increase in the buoyancy parameter Gr, the fluid velocity at surface, skin friction, and the rate of heat transfer increase. For the second solution, with the increase in Gr, the fluid velocity at surface decreases; the skin friction increases while the rate of heat transfer decreases. It is important to point that for $Gr = 1.0$, the second solution could not be numerically achieved. Table 2 also shows that with the increase in Pr, for first solution, the fluid velocity at surface and the skin friction decrease while the rate of heat transfer increases. For the second solution, with the increase in Pr, the fluid velocity at surface increases; the skin friction decreases while the rate heat transfer increases. For $Pr = 5.0$ and above, the solution could not be achieved.

Fig. 2a shows that for the both first solution and the second solution, the skin friction decreases with the increase in λ. In Fig. 2b, it is seen that for first solution the rate of heat transfer increases with the increase in λ, while for second solution the rate of heat transfer decreases with the increase in λ. Fig. 3a depicts that for the both first solution and second solution, the skin friction decreases with the increase in δ. In Fig. 3b, it is observed that for first solution and second solution, with the increase in δ, rate of the heat transfer decreases.

Fig. 4a, for the first solution, shows that for a lower value of S, the fluid velocity at surface is low, rises, and is greater in comparison with a higher value of S. It is also seen that with the increase in S, the velocity boundary layer thickness decreases, which is in agreement with natural phenomena. However, in the case of the second solution, the lower is the value of S, the

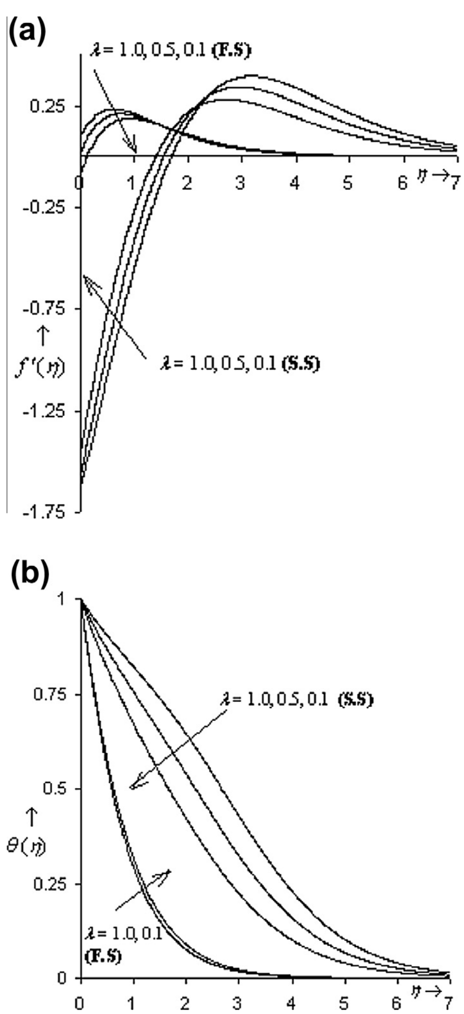

Figure 5 (a) Velocity distribution versus η when $Pr = 1.0$, $S = 1.0$, $\delta = -1.0$ and $Gr = 1.0$. (b) Temperature distribution versus η when $Pr = 1.0$, $S = 1.0$, $\delta = -1.0$ and $Gr = 1.0$.

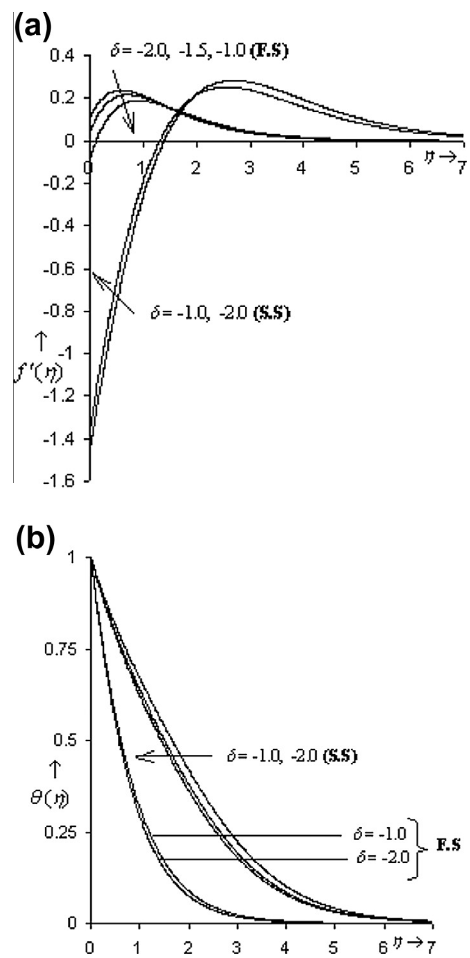

Figure 6 (a) Velocity distribution versus η when $Pr = 1.0$, $S = 1.0$, $\lambda = 0.1$ and $Gr = 1.0$. (b) Temperature distribution versus η when $Pr = 1.0$, $S = 1.0$, $\lambda = 0.1$ and $Gr = 1.0$.

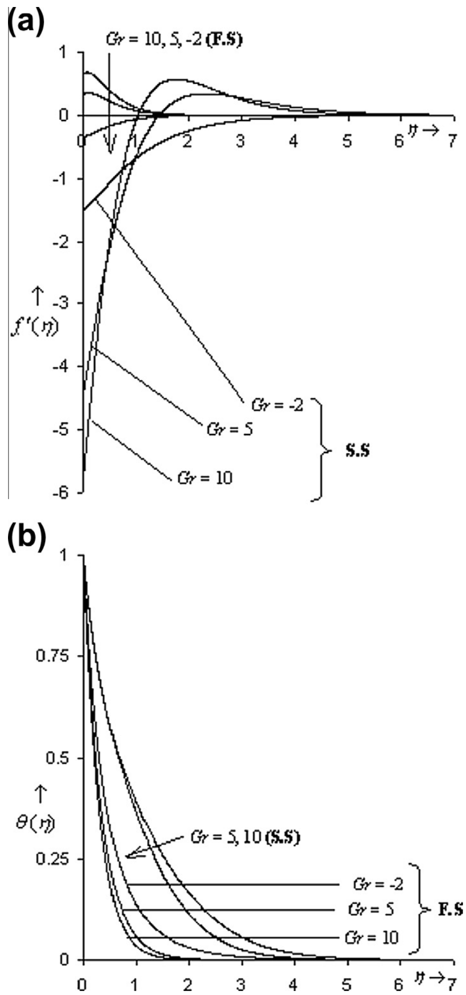

Figure 7 (a) Velocity distribution versus η when $Pr = 1.0$, $S = 3.0$, $\lambda = 0.1$ and $\delta = -1.0$. (b) Temperature distribution versus η when $Pr = 1.0$, $S = 3.0$, $\lambda = 0.1$ and $\delta = -1.0$.

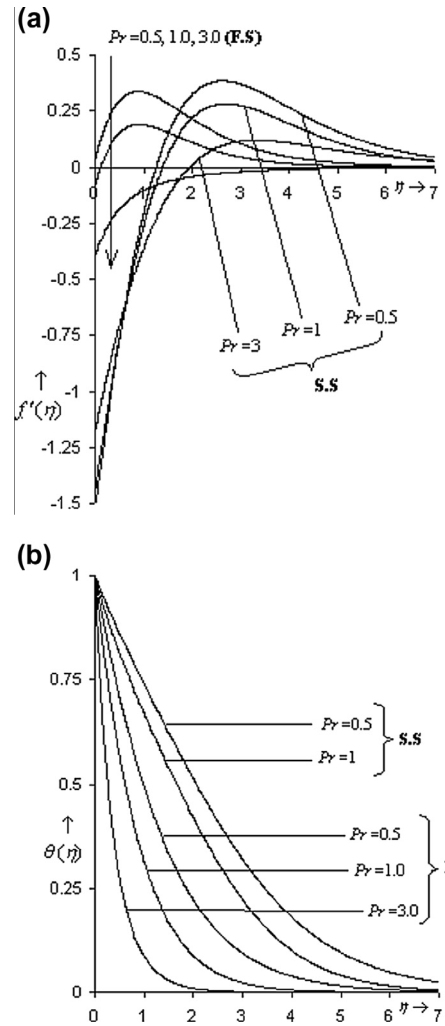

Figure 8 (a) Velocity distribution versus η when $\lambda = 0.1$, $S = 1.0$, $\delta = -1.0$ and $Gr = 1.0$. (b) Temperature distribution versus η when $\lambda = 0.1$, $S = 1.0$, $\delta = -1.0$ and $Gr = 1.0$.

higher is the fluid velocity. The boundary layer thickness remains almost the same with the change in S. It is seen from Fig. 4b that the fluid temperature decreases with the increase in S for both solutions. It is also seen that for the first solution, the thermal boundary layer thickness decreases with the increase in S while it remains almost the same for the second solution. Comparing the first solution and the second solution in Fig. 4a and b, the second solution has a thicker boundary layer.

In Fig. 5a, it is seen that with the increase in λ, the fluid velocity increases for the first solution. In the case of the second solution, with the increase in λ, the fluid velocity decreases but as η increases, the velocity profiles intersect and a reverse criterion is observed. Thus, no fixed trend is observed. Fig. 5b shows that for the first solution, the fluid temperature decreases slightly with the increase in λ, while for the second solution, with the increase in λ, the fluid temperature increases.

Fig. 6a depicts that with the increase in δ, the fluid velocity decreases for the first solution. In the case of the second solution, the higher the value of δ, the lower is the fluid velocity but with the increase in η, the velocity profiles intersect and the behavior changes. Fig. 6b shows that with the increase in δ, the fluid temperature increases in the case of the first solution while decreases for the second solution. We would like to comment for the first solution that the fluid velocity increases while the fluid temperature decreases with the increase in λ and the decrease in δ, which is true since an increase in the fluid velocity leads to better convection.

The tendency of buoyancy is to increase the fluid velocity. Fig. 7a shows that in the case of the first solution, the fluid velocity increases with the increase in Gr. The second solution does not show any fixed criterion. It is observed from Fig. 7b that the fluid temperature decreases with the increase in Gr for both the first and the second solutions. Fig. 8a depicts for the first solution that with the increase in Pr, the fluid velocity decreases. However, for the second solution, as discussed above, no fixed criterion is observed. It is seen from Fig. 8b that with the increase in Pr, the fluid temperature decreases for both the first solution and the second solution.

5. Concluding remarks

The boundary layer flow and heat transfer along a vertical isothermal shrinking sheet with a second-order slip have been studied numerically. It is found that a dual solution exists and the solutions are numerically stable and asymptotic in nature. It is important to add that there exist cases where the second solution could not be achieved. The first solution, however, shows trends, which are in agreement with studies present in the literature and vindicates applicability of the Wu [3] model. The second solution shows different trends from the first solution and is of theoretical importance. It is seen that the velocity and thermal boundary layer thickness are greater in the case of the second solution. It is hoped that the present study would help in understanding the newly proposed Wu [3] model in the case of boundary layer flows.

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